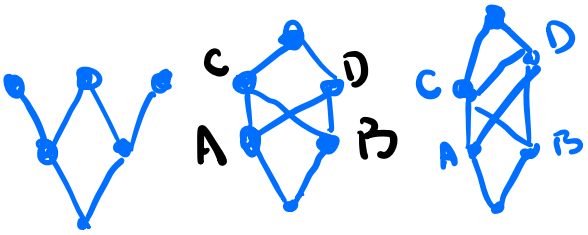
Bytecode

Instr $I ::= p = rv \mid \text{goto } i$

$\mid \text{if } p \text{ goto } i_1 \text{ else } i_2$

Place $p ::= x$

Rvalue $rv ::= c \mid p \mid p_1 \oplus p_2$



$A \sqcup B =$ _____

Lattices

Given a set $\mathcal{L}$, $\sqsubseteq$ is a partial order if

- reflexive: $x \sqsubseteq x$
- transitive: $x \sqsubseteq y \wedge y \sqsubseteq z \Rightarrow x \sqsubseteq z$
- anti-symmetric: $x \sqsubseteq y \wedge y \sqsubseteq x \Rightarrow x = y$

"join" (unique)

$x \sqcup y$ is the least upper bound of x and y.

i.e. the smallest z s.t. $x \sqsubseteq z \wedge y \sqsubseteq z$

$\mathcal{L}$ is a join-semilattice wrt $\sqsubseteq$ if

$\forall x, y \in \mathcal{L}.\ x \sqcup y$ exists

i.e. $\mathcal{L}$ contains a greatest element

1: $x = 10$

2: if $x \geq 0$ goto 3 else 6

3: $y = 1/x$

4: $x = x - 1$

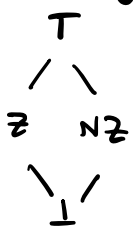
5: goto 2

6: exit

	x	y	x	y
σ_1^{in}	$\bot$	$\bot$	$\bot$	$\bot$
σ_1^{out}	NZ	$\bot$	NZ	$\bot$
σ_2^{in}	NZ	$\bot$	$\top$	$\top$
σ_2^{out}	NZ	$\bot$	$\top$	$\top$
σ_3^{in}	NZ	$\bot$	$\top$	$\top$
σ_3^{out}	NZ	$\top$	$\top$	$\top$
σ_4^{in}	NZ	$\top$	$\top$	$\top$
σ_4^{out}	$\top$	$\top$	$\top$	$\top$
σ_5^{in}	$\top$	$\top$	$\top$	$\top$
σ_5^{out}	$\top$	$\top$	$\top$	$\top$

$\mathcal{L} = \{\bot, \top, Z, NZ\}$

Hasse diagram:

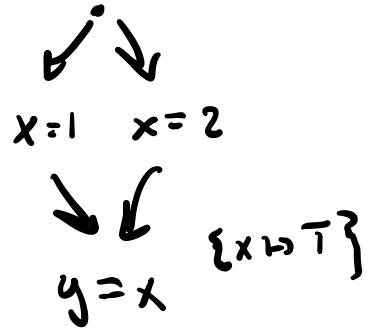


Constant analysis + propagation

$\{x \mapsto \bot, y \mapsto \bot\}$

$x = 1$ $\{x \mapsto 1, y \mapsto \bot\}$

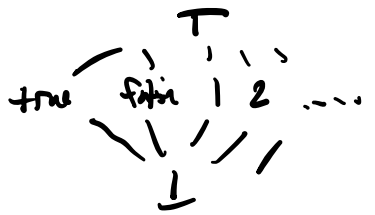
$y = x$ $\{x \mapsto 1, y \mapsto 1\}$



x=1
x=2
y=x

Abstract domain

$AbsConst = \{\bot, \top\} \cup Const$



$\sigma \in \{Var \mapsto AbsConst\}$

$\sigma_1 \cup \sigma_2 = \{x \mapsto \sigma_1[x] \sqcup \sigma_2[x]\}$

Transfer function

$f(x = c, \sigma) = \sigma[x \mapsto c]$

$f(x = y, \sigma) = \sigma[x \mapsto \sigma[y]]$

$f(x = x_1 \oplus x_2, \sigma) = \sigma[x \mapsto \top]$

$f(\text{if } x \text{ goto } i_1 \text{ else } i_2) = \sigma$

if $x = 0$ goto i_1 else i_2

$f(x = y - y, \sigma) = \sigma[x \mapsto 0]$

Liveness

$\sigma \in Set[Var]$

$\{\}$

$x = 0$ $\{x\}$

$y = x + 1$ $\{y\}$

$z = y - 2$ $\{z, y\}$

$w = y * z$ $\{\}$

$f(x, \sigma) = \sigma \cup used(x) - def(x)$

$def(x = \_) = \{x\}$

$used(x = y \oplus z) = \{y, z\}$

Backwards dataflow

$\sigma_i^{in} = f(p_i, \sigma_i^{out})$

$\sigma_i^{out} = \bigsqcup_{j \in succ(i)} \sigma_j^{in}$