

$$\begin{array}{l} \text{let } x: _ = 1 \text{ in} \\ \text{let } y: _ = x \text{ in} \end{array} \quad \rightarrow \quad \begin{array}{l} x: 2.0 = 1 \\ y: 2.1 = x \end{array}$$
$$x: 20 = 1$$

$$y: 21 = x$$
$$f_n f(x; -) \Rightarrow \{$$

3

$$2,0 = 21$$

Constraints:

? 0 = int

$$21 = 20$$

Goal:

mapping from
holes to concrete types

$$\{ ?0 \mapsto \text{int}, ?1 \mapsto \text{int} \}$$

Union-find

find(x) returns rep. of x
union(x, y) says $x = y$

find $(A) = A$

$$\text{find}(A) = C$$
$$\text{Find}(A) = C$$
$$f_{ind}(A) = E$$

?0 → ?1 → int

string $\rightarrow ?2$

union (?.0, ?.1)

union(int, int)

union(string, ?2)

$$\text{find}(??) = ??$$

find(?) = int

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fint(int) = int
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find(string) = 3.2

$\text{unify}(\tau_1, \tau_2):$
 $\tau_1 = \text{normalize}(\tau_1)$
 $\tau_2 = \text{normalize}(\tau_2)$
 $\text{match}(\tau_1, \tau_2):$

$\{ (\tau_n, \tau) \mid (\tau, \tau_n) \Rightarrow$
 $\text{union}(\tau_n, \tau)$

$(\text{int}, \text{int}) \Rightarrow \checkmark$

$(\text{int}, \text{bad}) \Rightarrow \times$

$((\tau_1, \dots, \tau_n), (\tau'_1, \dots, \tau'_n)) \Rightarrow$
 $\text{unify}(\tau_1, \tau'_1) \dots \text{unify}(\tau_n, \tau'_n)$

$([\tau], [\tau']) \Rightarrow$
 $\text{unify}(\tau, \tau')$

$\text{unify}((\tau_1, \tau_2), \tau_0)$

$\{ \tau_0 \mapsto (\tau_1, \tau_2)$
 $\tau_1 \mapsto \text{int} \}$

$\text{normalize}([\tau_0])$
 $= [(\text{int}, \tau_2)]$

$(\tau_0, \tau_1) = (\text{int}, \tau_2)$

$\tau_2 = \text{string}$

$\text{normalize}(\tau):$
 replace every hde in
 τ with $\text{find}(\text{hole})$
 (recursively)

$\tau_0 = (\tau_1, \tau_2)$

$\tau_0 = \text{int}$

$\text{unify}(\tau_0, (\tau_1, \tau_2))$

$\{ \tau_0 \mapsto (\tau_1, \tau_2) \}$

$\text{unify}(\tau_0, \text{int})$

$\text{normalize}(\tau_0) = (\tau_1, \tau_2)$

$\text{normalize}(\text{int}) = \text{int}$

$\text{unify}([?0], ?1).$

$\text{union}(?1, [?0])$

$\{ ?1 \mapsto [?0] \}$

$\text{unify}(?1, [\text{int}]).$

$\text{normalize}(?1) = [?0]$

$\text{unify}([?0], [\text{int}])$

$\text{unify}(?0, \text{int})$

$\text{union}(?0, \text{int})$

$\{ ?1 \mapsto [?0], ?0 \mapsto \text{int} \}$

$\text{unify}(?2, (?1, ?0)).$

$\text{normalize}((?1, ?0)) = ([\text{int}], \text{int})$

$\text{union}(?2, ([\text{int}], \text{int}))$

$\text{normalize}(?2) = ([\text{int}], \text{int})$

$\{ ?1 \mapsto [?0], ?0 \mapsto \text{int} \}$

$?2 \mapsto ([\text{int}], \text{int}) \}$

$\text{let } x \stackrel{?0}{=} ([?]) \text{ in}$

$\vdots$

$x := [1]$

$[?] : [?n]$

$?0 = [?n]$

$?0 = [\text{int}]$

$\text{let } x : _ = (1, 2) \text{ in}$

$\text{let } y \stackrel{?1}{=} (x, 0) :$

$?0 = (\text{int}, \text{int})$

$?1 = ?0.\emptyset$

$\uparrow_1 = \uparrow_2$

$\nwarrow$
not a type

$$\text{type}(x) = (?1, ?2)$$

$$?1 = ?2$$

Direct constraint

$$?0 = (?1, ?2)$$

$$?3 = ?0.0$$

$$?4 = ?3.0$$

$$?1 = (\text{int}, \text{int})$$

$$?2 = \text{string}$$

Proj. constraints

$$\begin{aligned} &\text{let } x : ?1 = 1 \\ &\rightarrow \text{let } x : ?2 = "hi" \\ &\rightarrow \end{aligned}$$

Projection constraints:

$$\tau_1 = \tau_2.i$$

$$\begin{aligned} \{ '0 &\mapsto ('1, '2) \\ '1 &\mapsto (\text{int}, \text{int}), \\ '2 &\mapsto \text{string} \} \end{aligned}$$

$$\text{normalize}('3) = '3$$

$$\text{normalize}('0) = ((\text{int}, \text{int}), \text{string})$$

$$?3 = (\text{int}, \text{int})$$

$$\text{unify}(?3, (\text{int}, \text{int}))$$

$$\text{normalize}('3) = (\text{int}, \text{int})$$

$$\text{unify}(?4, \text{int})$$