Expr $e ::=$
c
$\mid e_1 \oplus e_2$
$\mid \text{let } x = e_1 \text{ in } e_2$
$\mid x$

Type $\tau ::=$ int | string

Binop $\oplus ::= + \mid \wedge$

$\boxed{e \hookrightarrow e'}$

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{e_1 \text{ val}}{\text{let } x = e_1 \text{ in } e_2 \hookrightarrow e_2[x \mapsto e_1]}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

$$\frac{\Gamma(x)}{\Gamma \vdash}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2} \text{ T-Let}$$

$$\frac{e_1 \hookrightarrow e_1'}{\text{let } x = e_1 \text{ in } e_2 \hookrightarrow \text{let } x = e_1' \text{ in } e_2}$$

Progress:

if $\cdot \vdash e : \tau$ then e val
or $e \hookrightarrow e'$

Preservation:

if $\cdot \vdash e : \tau$ and
$e \hookrightarrow e'$ then
$\cdot \vdash e' : \tau$

Case T-Let:

$$\frac{\cdot \vdash e_1 : \tau_1 \quad x : \tau_1 \vdash e_2 : \tau_2}{\cdot \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2}$$

- By IH,

$\cdot \vdash e_1 : \tau_1$ then e_1 val,
or $e_1 \hookrightarrow e_1'$

- By T-Let assumption, $\cdot \vdash e_1 : \tau_1$ so e_1 val or $e_1 \hookrightarrow e_1'$.

- If e_1 val:
  then (let ...) $\hookrightarrow e_2[x \mapsto e_1]$
- $e_1 \hookrightarrow e_1'$
  then (let ...) $\hookrightarrow$ let $x = e_1'$ in e_2

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2}\ \text{T-let}$$

$$\frac{e_1 \text{ val}}{\text{let } x = e_1 \text{ in } e_2 \hookrightarrow e_2[x \mapsto e_1]}\ \text{E-Let-Val}$$

$$\frac{e_1 \hookrightarrow e_1'}{\text{let } x = e_1 \text{ in } e_2 \hookrightarrow \text{let } x = e_1' \text{ in } e_2}\ \text{E-Let-Step}$$

If $\bullet \vdash e : \tau$ and $e \hookrightarrow e'$ then $\bullet \vdash e' : \tau$

$$\frac{\bullet \vdash e_1 : \tau \qquad x_1 : T_1 \vdash e_2 : \tau_2}{\bullet \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_1}$$

- Case T-Let.
  - Case E-Let-Step.
    have $e_1 \hookrightarrow e_1'$.

WTS $\cdot \vdash \text{let } x = e_1' \text{ in } e_2 : \tau_2$

By IH on e_1, $\cdot \vdash e_1' : \tau_1$

By T-let, $\cdot \vdash \text{let } x = e_1' \text{ in } e_2 : \tau_2$.

$\boxed{\text{Lemma}}$

if $\Gamma \vdash e : \tau$

and $\Gamma(x) = \tau'$

and $\Gamma \vdash e' : \tau'$

then $\Gamma \vdash e[x \mapsto e'] : \tau$

- Case E-let-Val:

have e_1 val

WTS $\cdot \vdash e_2[x \mapsto e_1] : \tau_2$

By subst. lemma, this is true.